Year 11 Mathematics Methods AEMAM Term 1 2022

Test 1 Counting and Probability

Calculator Free
Formula Sheet Allowed

Student Name: Solutions

Teacher: Cheshire Feutrill Loh McRae

Time Allowed: 30 minutes

Calculator Free	/32
Calculator Assumed	/22
Total	/54

*Attempt **all** questions.*
*All necessary working and reasoning must be shown for **full marks**.*
Marks may not be awarded for untidy or poorly arranged work.

Question 1 [5 marks: 2, 1, 2]

a) Evaluate $\binom{10}{2} \times \binom{8}{6}$. $= \frac{10!}{2!\cancel{8!}} \times \frac{\cancel{8!}}{6!\,2!}$

$= \frac{10 \times 9 \times \overset{2}{\cancel{8}} \times 7}{\cancel{2} \times \cancel{2}}$

$= 1260$

✓ expands $\binom{n}{r}$ correctly
✓ evaluates

b) Given $\binom{n}{a} = \binom{n}{b}$, where $a \neq b$,
i) explain why $a + b = n$, with reference to Pascal's Triangle.

Each row of Pascal's Triangle is symmetrical. Since choosing r objects leaves $n-r$ objects and vice versa, therefore $a+b=n$

✓ provides any reasonable explanation w.r.t. Pascal's △

ii) hence or otherwise, determine the value of h.

$$\binom{25}{2h} - \binom{25}{h-2} = 0$$

$2h + h - 2 = 25$
$3h - 2 = 25$
$h = 9$

✓ recognises $2h + h - 2 = 25$
✓ evaluates h

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Question 2 [7 marks: 1, 1, 1, 1, 1, 2]

Given
- The universal set, $U = \{x: x \in \mathbb{Z}, 1 \leq x \leq 20\}$, where $\mathbb{Z}$ denotes the set of all integers
- Set $M = \{x: x\ is\ a\ multiple\ of\ 6\}$
- Set $L = \{\,x: x\ is\ a\ factor\ of\ 72\}$

a) Using set notation, list the elements of

i) L

$L = \{1, 2, 3, 4, 6, 8, 9, 12, 18\}$ ✓ lists elements of L

ii) M

$M = \{6, 12, 18\}$ ✓ lists elements of M

b)

i) Determine $n(M \cap L')$.

$n(M \cap L') = 0$ ✓ determines $n(M \cap L')$

ii) Hence, explain the relation between set M and set L.

M is a subset of L or $M \subset L$

✓ states $M \subset L$

c) If a number is chosen randomly from U, determine:

i) $P(\overline{M \cup L})$

$P(\overline{M \cup L}) = \frac{11}{20}$

✓ states probability

ii) $P(M|L)$

$P(M|L) = \frac{3}{9}$ ✓ uses correct sample size

✓ states probability

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Question 3 [3 marks]

Given $P(B|A) = \frac{1}{3}$ $P(B|A') = \frac{3}{4}$ and $P(A) = \frac{3}{5}$

Determine $P(B')$.

$P(B|A) = \frac{P(A \cap B)}{P(A)}$

$P(A \cap B) = \frac{3}{5} \times \frac{1}{3}$

$= \frac{1}{5}$ ✓

determines $P(A \cap B)$

$P(B|A') = \frac{P(B \cap A')}{P(A')}$

$P(B \cap A') = \frac{2}{5} \times \frac{3}{4}$

$= \frac{3}{10}$ ✓

determines $P(B \cap A')$

$P(B) = P(A \cap B) + P(B \cap A')$

$= \frac{1}{5} + \frac{3}{10}$

$= \frac{5}{10}$

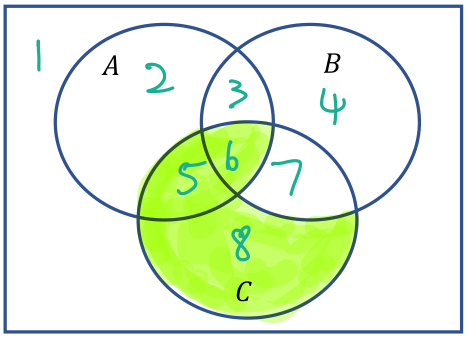
$P(B') = 1 - P(B)$

$= \frac{5}{10}$

✓ determines $P(B')$

Question 4 [4 marks: 1, 2, 1]

a) Shade $A \cup \bar{B} \cap C$.



A: Areas 2, 3, 5, 6

$\bar{B}$: Areas 1, 2, 5, 8

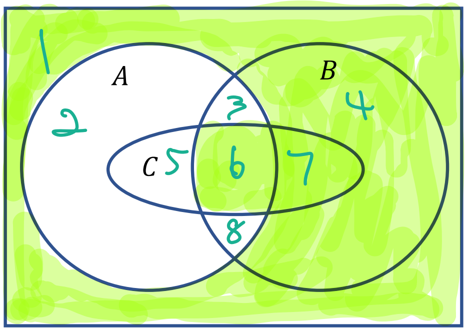
$A \cup \bar{B}$: Areas 1, 2, 3, 5, 6, 8

C: Areas 5, 6, 7, 8

$\therefore A \cup \bar{B} \cap C$: Areas 5, 6, 8

✓ shades areas correctly

b) Shade $(A \cup C)' \cup (B \cap C)$.



A: Areas 2, 3, 5, 6, 8

C: Areas 5, 6, 7

$(A \cup C)'$: Areas 1, 4

B: Areas 3, 4, 6, 7, 8

C: Areas 5, 6, 7

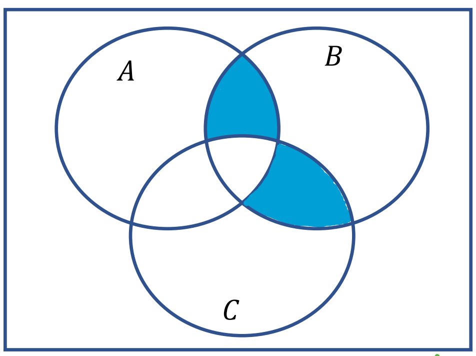
$(B \cap C)$: Areas 6, 7

$(A \cup C) \cup (B \cap C)$ = Area 1, 4, 6, 7

✓✓ shades area correctly

-1 / missed area

c) Use set notation to describe the shaded region in the Venn diagram below.



$(A \cap B \cap C') \cup (B \cap C \cap A')$

✓ describes shaded area correctly using set notation

Other possible solutions: $(A \cap B) \cup (B \cap C) \cap (\overline{A \cap C})$

$(B \cap C) \cup (A \cap B) \cap (\overline{A \cap B \cap C})$

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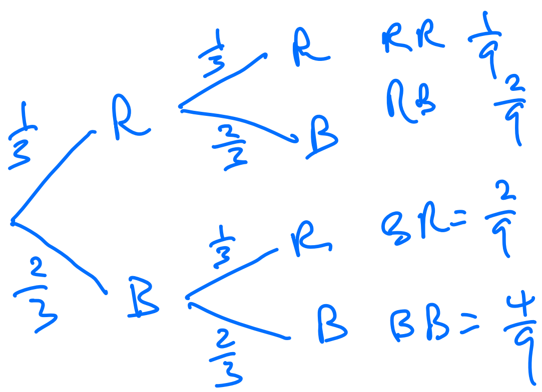
Question 5 [6 marks: 4, 2]

In the Game of Untouchable, a player has two spins of a spinner with two colours, red and black. The first person to spin two successive different colours (red followed by black or vice versa) wins the game. In the instruction manual, it is stated that the probability of spinning 2 successive reds from the spinner is $\frac{1}{9}$.

a) Determine the probability of spinning two successive different colours.



$P(R) \times P(R) = \frac{1}{9}$ ✓ recognizes $P(R)^2 = \frac{1}{9}$

$\therefore P(R) = \frac{1}{3}$ ✓ determines $P(R)$

$P(B) = \frac{2}{3}$ ✓ determines $P(B)$

$P(RB \text{ or } BR) = \frac{2}{9} + \frac{2}{9}$

$= \frac{4}{9}$ ✓ determines $P(RB \text{ or } BR)$

b) James was given the game as a Christmas present. Without reading the instruction manual, he claims that the chance of him winning the game with his first two spins is 50/50. Do you agree with his statement? Justify your answer with appropriate calculations.

$P(\text{winning}) = P(RB \text{ or } BR) = \frac{4}{9}$ ✓ Shows appropriate calculation

$\frac{4}{9} < \frac{1}{2}$

Since probability of winning is $< \frac{1}{2}$, James' statement is incorrect. ✓ states conclusion with reasons

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Question 6 [7 marks: 3, 2, 2]

a) Expand $\left(1+\frac{x}{4}\right)^4$.

$$\left(1+\frac{x}{4}\right)^4 = 1^4 + 4(1)^3\left(\frac{x}{4}\right) + 6(1)^2\left(\frac{x}{4}\right)^2 + 4(1)\left(\frac{x}{4}\right)^3 + \left(\frac{x}{4}\right)^4$$

✓ ✓ uses correct Pascal's △ / expansion method

$$= 1 + 4\left(\frac{x}{4}\right) + 6\left(\frac{x^2}{16}\right) + 4\left(\frac{x^3}{64}\right) + \frac{x^4}{256}$$

$$= 1 + x + \frac{3x^2}{8} + \frac{x^3}{16} + \frac{x^4}{256}$$

✓ expands

b) Hence, show that $\left(1+\frac{x}{4}\right)^4 - \left(1-\frac{x}{4}\right)^4 = 2x + \frac{x^3}{8}$.

$$\left(1-\frac{x}{4}\right)^4 = 1 - x + \frac{3x^2}{8} - \frac{x^3}{16} + \frac{x^4}{256}$$

✓ states terms of $\left(1-\frac{x}{4}\right)^4$

$$\left(1+\frac{x}{4}\right)^4 - \left(1-\frac{x}{4}\right)^4 = \cancel{1} + x + \cancel{\frac{3x^2}{8}} + \frac{x^3}{16} + \cancel{\frac{x^4}{256}} - \cancel{1} + x - \cancel{\frac{3x^2}{8}} + \frac{x^3}{16} - \cancel{\frac{x^4}{256}}$$

$$= 2x + \frac{2x^3}{16}$$

$$= 2x + \frac{x^3}{8}$$

✓ shows steps of simplification

c) Determine the exact value of $(1.25)^4 - (0.75)^4$.

$1.25 = 1 + \frac{1}{4}$ $\quad$ $0.75 = 1 - \frac{1}{4}$ $\quad$ $\therefore x = 1$ ✓ recognises $x = 1$

$$\therefore (1.25)^4 - (0.75)^4 = \left(1+\frac{1}{4}\right)^4 - \left(1-\frac{1}{4}\right)^4$$

$$= 2 + \frac{1}{8}$$

$$= \frac{17}{8}$$

✓ determines the exact value

End of Calculator Free Section

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Year 11 Mathematics Methods
AEMAM Term 1 2022

Test 1 Counting and Probability

Calculator Assumed
Formula Sheet, ClassPad and Calculator Allowed

Student Name: ____

Teacher: Cheshire Feutrill Loh McRae

Time Allowed: 20 minutes

Calculator Assumed:	/22

*Attempt **all** questions.*
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Question 7 [5 marks: 1, 2, 2]

A team of 6 players is to be selected from 10 volleyball players of whom 7 are from Group Shen and 3 are from Group Ton. If the selection of the team is made at random, determine

a) the number of ways the team can be selected.

$${}^{10}C_6 = 210$$

✓ determines the no. of ways the team can be selecting

b) the number of ways the team can be selected if we must have equal number of players from Group Shen and Group Ton.

$$\underbrace{{}^{3}C_3 \times {}^{7}C_3}_{\checkmark \text{ uses correct combinations}} = 35$$

✓ evaluates

c) the probability that at most two players from Group Ton are in the team.

$$\frac{210 - 35}{210} = \frac{175}{210}$$

✓ recognises complement
✓ states probability

OR

$$\frac{{}^{3}C_2 \times {}^{7}C_4 + {}^{3}C_1 \times {}^{7}C_5 + {}^{3}C_0 \times {}^{7}C_6}{210} = \frac{175}{210}$$

or
✓ uses appropriate combinations
✓ states probability

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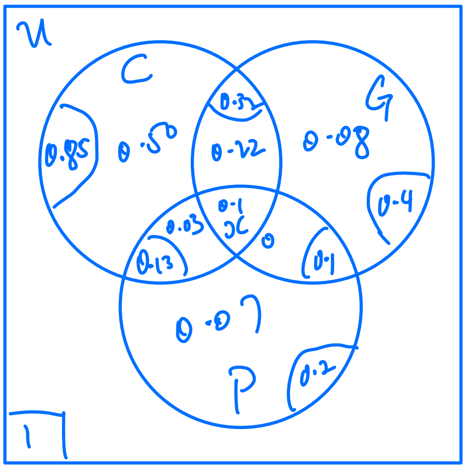
Question 8 [8 marks: 4, 2, 2]

In the Faculty of Ancient Languages in the University of Neverland, 85% of the students learn Coptic (C), 40% learn Gothic (G) and 20% learn Pali (P). 32% of the students learn Coptic and Gothic, 13% learn Coptic and Pali and 10% learn Gothic and Pali. All the students learn at least one of these three ancient languages.

a) Determine the percentages of students learning all three languages.



$1 = 0.85 + 0.4 + 0.2 - 0.32 - 0.13 - 0.1 + x$

$\therefore x = 0.1$ ✓ determines $P(C \cap P \cap G)$

10% of the students study all 3 languages

✓ state percentage.

✓ fills in correct totals for C, G & P

✓ fills in correct totals for all intersections.

b) If the number of enrolments in the faculty is 467, determine the number of students who study exactly two ancient languages.

% of students studying exactly 2 languages $= 0.22 + 0.03 + 0$

$= 0.25$ ✓ determine proportion

$0.25 \times 468 = 116.75$ ✓ calculate no. of students

117 students study exactly 2 languages studying exactly 2 languages.

c) Determine the probability of a student studying Pali given that the student did not study Coptic.

$P(P|C') = \dfrac{0.07}{0.07 + 0.08} = \dfrac{7}{15}$ ✓ states probability

✓ recognises sample space

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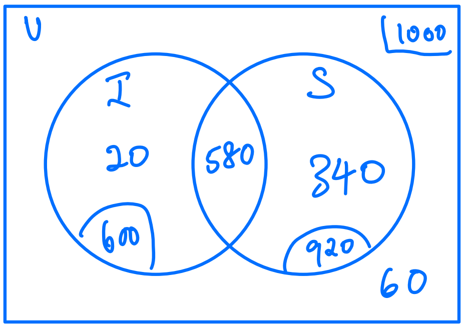
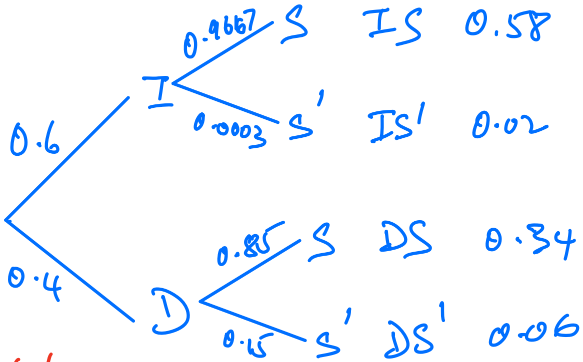
Question 9 [9 marks: 4, 2, 3]

Pink Panther Sports Company sells two brands of soccer ball, Ike and Didas. At the start of 2021, the store had 600 Ike soccer balls and 400 Didas soccer balls in stock. At the end of 2021, the store found that 80 of the soccer balls had not been sold. The store also found that the number of Ike soccer balls not sold was 20.

a) Determine the probability of a Didas ball not sold in 2021.



or



✓ draws Venn diagram
✓ shows $n(I \cap S')$, $n(U)$
✓ determines $n(I \cap S)$, $n(I' \cap S)$, $n(I \cup S)$

$P(D \text{ not sold}) = \frac{60}{1000}$

✓ determines probability

✓✓✓ draws tree diagrams with weighted branch, sample space and probability

$P(DS') = 0.06$

✓ determines probability

b) Of the soccer balls not sold, determine the probability of a ball being brand Ike.

$P(I | S') = \frac{20}{80}$ ✓ determines reduced sample space

✓ states probability

c) Is the sale of a soccer ball independent of the brand? Justify you answer.

$P(I) = \frac{600}{1000}$ $\quad P(S) = \frac{920}{1000}$

$P(I) \times P(S) = 0.552$ ✓ determines $P(I) \times P(S)$

$P(I \cap S) = \frac{580}{1000} = 0.58$ ✓ determines $P(I \cap S)$

Since $P(I) \times P(S) \neq P(I \cap S)$, the sale of a soccer ball is dependent on the brand

✓ states not independent with justification

Note: suggested independence Max 2 marks.

End of Calculator Assumed Section

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